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RHEOLOGICAL EQUATIONS OF STATE OF A DILUTE SUSPENSION OF DIPOLE DUMBBELLS IN A POWER-LAW LIQUID

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We obtain the rheological equations of state of dilute suspensions of dipole dumbbells in a power-law liquid. As an example, we consider simple shear flow of such a medium in an electric field.

In the present paper we consider a dilute suspension of rigid axially symmetric particles in a power-law liquid whose rheological equation of state has the form

$$\tau_{ij} = -p\delta_{ij} + 2m\,|2d_{km}d_{mk}|^{\frac{n-1}{2}}\,d_{ij}. \tag{1}$$

For n < 1 this model describes pseudoplastic liquids, while for n > 1 it describes dilatant liquids. The case n = 1 corresponds to Newtonian behavior.

We assume that the dispersion medium interacts with the suspended particles which have zero buoyancy as with hydrodynamic bodies. As a hydrodynamic model of the suspended particles, we consider the model of a rigid point dumbbell: At a distance L equal to the length of the real axially symmetric suspended particle there are point centers of hydrodynamic interaction of the model with the surrounding liquid. As for suspensions in Newtonian liquids, we assume that the ends of the dumbbell interact with a power-law dispersion medium like spheres of radius a.

We consider the Stokes approximation for flow past suspended particles. It is known that the coefficient of frictional resistance of a sphere of radius a moving with a translational velocity U is given in the Stokes approximation by the formula [1]:

$$\zeta = 4\pi\left(\frac{12}{n^2}\right)^{\frac{n+1}{2}} F(n)\,m\,U^{n-1}\,a^{2-n}. \tag{2}$$

The function F(n) has been tabulated in [2]; in particular, for n = 1 (a Newtonian liquid), $\zeta = 6\pi m a$.

The suspended axially symmetric particles may have a constant dipole moment $p_c = q\mathbf{n}$ or an induced dipole moment $p_e = \varkappa\,\mathbf{n}(\mathbf{E}\cdot\mathbf{n})$. The interaction between the electric fields of the suspended particles, like the hydrodynamic interaction between the particles, is disregarded.

Since the hydrodynamic model of the particles is so simple, the determining equation for the vector $\mathbf{n}$ characterizing the orientation of the dumbbell can be determined by using a structural approach.

Suppose that the length of the suspended dumbbells is such that the velocity of the dispersion medium within the limits of a particle is a homogeneous function of the coordinates. Then by the first Helmholtz theorem [3], the velocity of the liquid at the point where the

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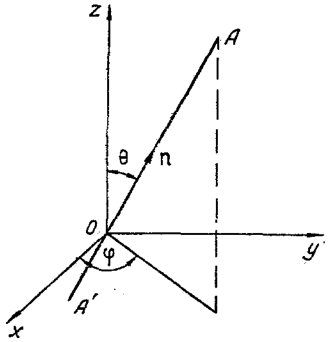

Fig. 1. Laboratory system of coordinates, the origin of which coincides with the midpoint of the L axis of the dumbbell AA'.

point center A of the hydrodynamic interaction of the dummbbell with the surrounding liquid is situated will be

$$v_i = \frac{L}{2} d_{ik} n_k + \frac{L}{2} \omega_{ik} n_k.$$

Since the frictional force resulting from flow past end A of the particle in the direction of the L axis is compensated by the action of the force applied to end A', which is symmetric with respect to the origin, it follows that the relative velocity v_{i0} of flow past end A of the dumbbell has the following components: In the direction of the L axis

$$\overset{*}{v}_{i0} = \frac{L}{2} d_{km} n_k n_m n_i \tag{3}$$

and perpendicular to that axis

$$v'_{i0} = \frac{L}{2} \ (d_{ik} n_k - d_{km} n_k n_m n_i - N_i), \tag{4}$$

where $N_i = \dot{n}_i - \omega_{ik} n_k$. The frictional forces resulting from the flow past the ends of the particle with velocity v'_{i0} give rise to the rotational moment $M_g = L[\mathbf{n} \times \mathbf{F}]$, where $F_i = \zeta v'_{i0}$ is the frictional force acting on one end of the particle. The resistance coefficient ζ, according to (2), will be $\zeta = \alpha(n) m a^{2-n} [(\overset{*}{v}_{i0})^2 + (v'_{i0})^2]^{\frac{n-1}{2}}$, or, taking account of (3), (4),

$$\zeta = \alpha(n) m a^{2-n} \left(\frac{L}{2}\right)^{n-1} [N_i N_i - 2 d_{ik} n_k N_i + d_{ik} d_{ij} n_k n_j]^{\frac{n-1}{2}}, \tag{5}$$

where

$$\alpha(n) = 4\pi \left(\frac{12}{n^2}\right)^{\frac{n+1}{2}} F(n).$$

After vector multiplication by n_i of the equation of motion of the suspended dumbbell

$$\frac{d\mathbf{L}}{dt} = \mathbf{M}_g, \tag{6}$$

where $\mathbf{L} = I[\mathbf{n} \times \dot{\mathbf{n}}]$, we obtain the determining equation for the vector n_i:

$$I(\ddot{n}_i + \dot{n}_k \dot{n}_k n_i) = \frac{1}{2} \zeta L^2 (d_{ik} n_k - d_{km} n_k n_m n_i - N_i). \tag{7}$$

If the suspended particles have a constant dipole moment $p_c = q\mathbf{n}$, then in the right side of Eq. (6), when there is an external uniform electric field with intensity E, we must take account of the moment of the electrical forces $M_c = q[\mathbf{n} \times \mathbf{E}]$ acting on the particle. In this case the determining equation for the vector n_i has the form

$$I(\ddot{n}_i + \dot{n}_k \dot{n}_k n_i) = \frac{1}{2} \zeta L^2 (d_{ik} n_k - d_{km} n_k n_m n_i - N_i) + q(E_i - n_k E_k n_i). \tag{8}$$

If the suspended particles represented by dumbbells in the model do not have a constant dipole moment but have dielectric susceptibility, then in Eq. (6) we must take account of the following moment of the electrical forces: $M_e = \varkappa(E \cdot n)[n \times E]$. The determining equation for n_i in this case will be

$$I(\ddot{n}_i + \dot{n}_k \dot{n}_k n_i) = \frac{1}{2} \zeta L^2 (d_{ik} n_k - d_{km} n_k n_m n_i - N_i) + \varkappa E_k n_k (E_i - n_k E_k n_i). \tag{8'}$$

In order to obtain the determining equation for the stress tensor in the medium under consideration, we shall make use of the structural-phenomenological approach of [5].

In choosing the phenomenological equation of state for the stress tensor, we make use of the fact that the expression for the rate of dissipation of energy per unit volume of the suspension is $E_s = E_p + [2N_0 \langle \zeta [(\overset{*}{v}_{i0})^2 + (v'_{i0})^2] \rangle$, where the averaging is carried out by means of the distribution function $F(n_i)$ for the axes of the particles according to angular position, determined from the equation

$$\frac{\partial F}{\partial t} + \frac{\partial}{\partial n_i}(F \dot{n}_i) = 0, \tag{9}$$

has the same form

$$E_s = E_p + N_0 \langle \frac{1}{2} \zeta L^2 (N_i N_i - 2 d_{ij} N_i n_j + d_{ij} d_{ik} n_j n_k) \rangle \tag{10}$$

for a Newtonian [4] and for a power-law dispersion medium. The difference lies in the fact that for a Newtonian dispersion medium the rheological diameter $W = {}^1/_2 \zeta L^2$ — the coefficient of rotational friction of the dumbbells — is a constant, while for a power-law dispersion medium, as can be seen from (5), it is a function of the invariants

$$N_i N_i, \; d_{ik} n_k N_i, \; d_{ik} d_{ij} n_k n_j. \tag{11}$$

Therefore the stress tensor in the suspension under consideration must be determined from a relation of the form $T_{ij} = \tau_{ij} + N_0 \langle t_{ij} \rangle$, $t_{ij} = t_{ij}(d_{km}, n_e, N_l)$, where the averaging is carried out with the aid of the distribution function $F(n_i)$, the solution of Eq. (9); τ_{ij}, as in a Newtonian dispersion medium [4], is a linear function of the tensor d_{km}, i.e.,

$$T_{ij} = \tau_{ij} + N_0 (\langle a_1 \rangle d_{ij} + \langle a_2 n_i n_j \rangle + d_{km} \langle a_3 n_k n_m n_i n_j \rangle +$$
$$+ d_{ik} \langle a_4 n_k n_j \rangle + d_{kj} \langle a_5 n_k n_i \rangle + \langle a_6 n_i N_j \rangle + \langle a_7 N_i n_j \rangle),$$

but the phenomenological parameters a_i (i = 1, 2, ..., 7) depend on the invariants (11).

The phenomenological rheological parameters a_i are found, as in [4], by a comparison of the rate of dissipation of energy per unit volume of suspension, determined in a manner analogous to [6] by the formula $E_s = T_{ij} d_{ij} - N_0 \langle g_i N_i \rangle$, with the expression (10) obtained on the basis of the structural theory, assuming that $T_{ji} - T_{ij} = N_0 \langle n_j g_i \rangle$ [6], where $g_i = {}^1/_2 \zeta L^2 (d_{ik} n_k - d_{km} n_k n_m n_i - N_i)$ is the right side of the determining equation (7). A check will show that the stress tensor obtained in this way,

$$T_{ij} = -p\delta_{ij} + 2m |2 d_{km} d_{mk}|^{\frac{n-1}{2}} d_{ij} + N_0 \langle \frac{1}{2} \zeta L^2 (d_{ik} n_k n_j - N_i n_j) \rangle, \tag{12}$$

has the same form if the orientation of the suspended particles is determined not only by the hydrodynamic forces (7) but also by the electrical field (8), (8').

As an example, we can consider the simple shear flow

$$v_x = 0, \; v_y = Kx, \; v_z = 0, \; K = \text{const} \tag{13}$$

of a medium with dipole particles when there is an electric field with intensity

$$E_x = E, \; E_y = 0, \; E_z = 0, \; E = \text{const}, \tag{14}$$

neglecting the moment of inertia of the suspended particles (I = 0).

Passing to spherical coordinates $n_x = \cos \varphi \sin \Theta$, $n_y = \sin \varphi \sin \Theta$, $n_z = \cos \Theta$ (Fig. 1), we find from (8) that the components of the angular velocity of the particle $\omega(\dot{\varphi}, \dot{\Theta})$ are determined by the relation

$$\dot{\varphi} = K \cos^2 \varphi - \frac{qE}{W} \cdot \frac{\sin \varphi}{\sin \Theta}, \tag{15}$$

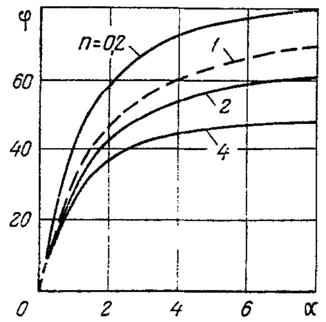

Fig. 2. "Hovering" angle φ, in degrees, of the dumbbell as a function of α (the dumbbell "hovers" in the shear plane). The dashed curve corresponds to a Newtonian dispersion medium (n = 1).

$$\dot{\Theta} = \frac{K}{4}\sin 2\varphi \sin 2\Theta + \frac{qE}{W}\cos\varphi\cos\Theta. \tag{16}$$

The solution of Eq. (15) for E = 0 is $\varphi = \arctan(Kt + C)$. For E = 0 we divide Eq. (15) by (16) and integrate to obtain $(\cos\varphi)^{-1} = C_1 \tan\Theta$. From the solutions obtained it follows that as $t \to \infty$, $\varphi \to \pi/2$, $\Theta \to \pi/2$, i.e., when there is no electric field, the motion of the suspended dumbbell is not periodic, and for sufficiently large values of t it will be oriented along the axis Oy.

A constant electric field with the intensity vector (14) will bring the particle out of this situation; it will take on an angular position in the shear plane such that the moment of the electric field will be equal to the moment of the frictional forces acting on the particle.

This analysis enables us to conclude that Eqs. (15) and (16) have a stationary solution when $E \neq 0$. Since for $E \neq 0$ these equations are nonlinear, it follows that the stationary angular position of the particle (as $t \to \infty$) can more conveniently be found not from the general solution but by setting $\dot{\varphi} = \dot{\Theta} = 0$. We find that a suspended particle is oriented in the plane xOy ($\Theta = \pi/2$) at an angle φ with respect to the axis Ox determined by the equation

$$\alpha(\cos^2\varphi)^{\frac{n+1}{2}} - \sin\varphi = 0, \tag{17}$$

where

$$\alpha = \frac{2\alpha(n)\, m\left(\frac{L}{2}\right)^{n+1} a^{2-n}\, |K|^{n-1} K}{qE}$$

is a dimensionless parameter. For n = 1 (a Newtonian dispersion medium) $\alpha = WK/qE$, and Eq. (17) can be solved analytically: $\varphi = \arcsin\,(1/2\alpha)(-1 + \sqrt{1 + 4\alpha^2})$; for n < 1 and n > 1 we can solve it numerically (Fig. 2).

From the foregoing results it can be seen that since the suspended dipole particles, irrespective of their initial orientation in stationary flow and under the action of a stationary electric field, take on the same constant angular position as $t \to \infty$; it follows that in determining the steady-state stress condition in the suspension in this case we do not need to carry out the averaging in (12) with respect to the angular positions of the particles.

From (12) and (5), for (13), (14) when $\dot{\varphi} = \dot{\Theta} = 0$, $\Theta = \pi/2$, we find

$$T_{(xy)} = \frac{1}{2}(T_{xy} + T_{yx}) = m\left[1 + N_0\alpha(n)\left(\frac{L}{2}\right)^{n+1} a^{2-n}(\cos^2\varphi)^{\frac{n+1}{2}}\right]|K|^{n-1}K, \tag{18}$$

$$T_{xx} - T_{zz} = 0, \tag{19}$$

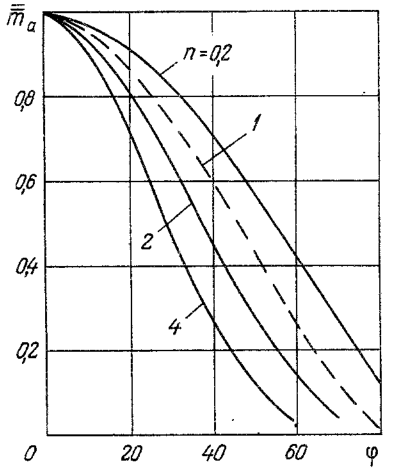

Fig. 3

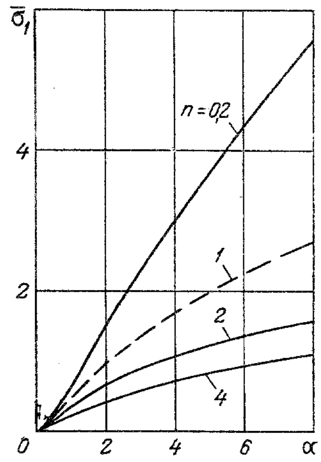

Fig. 4

Fig. 3. Variation of $\bar{m}_\alpha = (m_\alpha - m)(1/N_0\alpha(n)(L/2)^{n+1} \cdot a^{2-n})$ as a function of φ, deg.

Fig. 4. Variation of $\bar{\sigma}_1 = (T_{yy} - T_{zz})(1/N_0Eq)$ as a function of α.

$$T_{yy} - T_{zz} = 2N_0\alpha(n)\,ma^{2-n}\left(\frac{L}{2}\right)^{n+1}(\cos^2\varphi)^{\frac{n-1}{2}}\cos\varphi\sin\varphi\,|K|^{n-1}K. \tag{20}$$

From (18) it follows that the medium under consideration in the flow (13) with the field (14) behaves like a quasi-power-law liquid with an index of non-Newtonian behavior of the dispersion medium and an effective consistency

$$m_a = m\left[1 + N_0\alpha(n)\left(\frac{L}{2}\right)^{n+1}a^{2-n}(\cos^2\varphi)^{\frac{n+1}{2}}\right], \tag{21}$$

dependent not only on the parameters characterizing the medium (m, n, L, a, q, N_0) but also on the shear rate and the value of the intensity vector of the electric field.

The solutions of Eq. (17) (Fig. 2) show that for some finite E, when there is no flow ($K = 0$), the suspended particles are oriented parallel to Ox. As the shear rate increases, the particles rotate in the plane xOy, becoming oriented parallel to the axis Oy as $K \to \infty$.

This means that, as can be seen from Fig. 3, the increment of the effective consistency $m_\alpha - m$ of the medium resulting from the presence of suspended particles decreases as the shear rate increases.

As can be seen from formula (20), the medium displays the Weissenberg effect (Fig. 4).

In the absence of appropriate experimental data on suspensions in non-Newtonian liquids, we can only say that the nature and direction of the rheological behavior of dilute suspensions of rigid axially symmetric particles in a power-law and a Newtonian dispersion medium are analogous and differ only quantitatively.

The fact that a point dumbbell has no volume means, as in the case of a Newtonian dispersion medium, that for suspensions of dumbbells [7, 8] and rods (the "pearl necklace" model) [9-11], in stationary shear flow there will be a decrease in the values of $m_\alpha - m$ and $\mu_\alpha - \mu_p$ as $K \to \infty$. This defect is absent when we have an ellipsoid of revolution, but when we use this as a model for the suspended particles in a power-law liquid, we find that when we obtain the rheological equations of state, we have partial differential equations, which cannot be solved analytically because they are strongly nonlinear.

NOTATION

τ_{ij}, stress tensor in power-law liquid; p, pressure; δ_{ij}, Kronecker delta; d_{ij}, deformation rate tensor; m, index of consistency of power-law liquid; n, index of non-Newtonian behavior of power-law liquid; q, value of constant dipole moment; **n**, unit vector characterizing the orientation of a suspended axially symmetric particle in the laboratory rectangular Cartesian coordinate system x, y, z with origin at the midpoint of the axis of symmetry; $\varkappa$, principal value of the dielectric susceptibility of the axially symmetric particle along the axis of symmetry; **E**, intensity vector of the electric field; ω_{ik}, velocity vortex tensor; $\dot{\mathbf{n}}$, $\dot{n}_i$, and $\ddot{n}_i$, $\dot{\varphi}$, $\dot{\Theta}$, derivatives with respect to time; L, moment of inertia of dumbbell; I, moment of inertia of dumbbell with respect to the axis passing through the midpoint of the particle perpendicular to it; $\langle\ \rangle$, symbol indicating averaging by means of distribution function; $E_p=2m|2d_{km}d_{mk}|^{\frac{n-1}{2}}d_{ij}d_{ij}$, rate of dissipation of energy per unit volume of power-law dispersion medium in the absence of suspended particles; N_0, number of suspended particles per unit volume of suspension; $N_0\langle t_{ij}\rangle$, stress produced by the presence of N_0 suspended particles per unit volume of suspension; K, shear rate in simple shear motion; E, value of intensity vector of electric field; W, coefficient of rotational friction of the dumbbell; C, C_1, constants of integration; t, time; $\mu_a = (1/K)T_{(xy)}$, effective viscosity of suspension in simple shear flow; $\mu_p = m|K|^{n-1}$, viscosity of power-law dispersion medium in simple shear flow.

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RELATION BETWEEN HOMOGENEOUS AND INHOMOGENEOUS STRETCHING OF AN ELASTIC FLUID

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Homogeneous and inhomogeneous (steady) noninertial stretching of an elastic fluid is experimentally investigated, and a method is given for the calculation of one problem from experimental data on the other.

1. Homogeneous Stretching with Constant Force

The noninertial homogeneous stretching of cylindrical samples was first studied experimentally in [1, 2]. The experimental arrangement for stretching of this type is shown in Fig. 1a. One end of the test sample is fixed, and the other moves under the action of a constant force F. In the cylindrical coordinate system x, r, φ, the velocity components are

$$v_x = \varkappa(t)x;\ v_r = \frac{\varkappa(t)}{2}r;\ v_\varphi = 0. \tag{1.1}$$

Here x is the longitudinal coordinate measured from the point of fixing of the sample (Fig. 1a).

On the basis of the incompressibility condition for the fluid, the sample radius is

$$r(t) = r_0\varepsilon^{-1/2};\ \varepsilon = l/l_0. \tag{1.2}$$

Here r_0 and l_0 are the initial (t = 0) sample radius and length. Under the condition that the radial component of the stress tensor vanishes, and taking into account Eq. (1.2), the stress in the sample cross section is

$$\sigma = \sigma_{xx} = F/\pi r^2 = \sigma_0\varepsilon;\ \sigma_0 = F/\pi r_0^2. \tag{1.3}$$

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